<u>AC Circuits:</u>

This chapter deals with circuits that have an AC voltage source (such as a generator) and a combination of resistors (R), inductors (L), and capacitors (L). We need to make a few **conventions:**

Since the applied voltage alternates, the current in the circuit and the voltages across the various components of the circuit (R, L, or C) will also depend on time. We use **lower-case letters** (i,v,p) to denote **instantaneous** values and **upper-case letters** for **time-averaged** or **rms** values (I, V, P) or for **constants** that do not change with time (R, L, C). A voltage with a **subscript** tells us the voltage drop **across a component**. A current with a subscript is the current through a component. The subscript 0 (as in V_0) is the **amplitude** (or **peak value**) of the time-dependent quantity. For example:

v_R instantaneous voltage across the resistor R
v_C instantaneous voltage across the capacitor C
v_L instantaneous voltage across the inductor L
v applied voltage (from the AC voltage source)

We assume that the current i is zero at time $t = 0$, therefore the current is always given by $i(t) = i_0 \sin(wt)$. Then, the applied voltage is $v(t) = v_0 \sin(wt + j)$, since there may be a phase difference j between the current and the voltage.

We will need to calculate average quantities, for example an average current defined by $I_{\text{av}} = \lim_{T\to\infty} \frac{1}{T}\int_0^T i(t)dt$. It is usually sufficient to integrate over a complete AC cycle.

<u>A resistor R in an AC Circuit:</u>

The current through the resistor R is $i_R(t) = i_0 \sin(wt)$ by definition. According to Ohm's law, the voltage drop across R is $v_R(t) = i_R(t)R = v_{0R} \sin(wt)$, where $v_{0R} = i_0 R$. According to Kirchhoff's loop rule, the external AC voltage required to drive this current is $v = v_{0R} \sin(wt)$.

We see that in a circuit containing only a resistor R, the current and voltage are **<u>in phase</u>**. The phase lag Φ is zero.

The power dissipated in the resistor at each instant is

$$p(t) = i(t)v(t) = i_0 \sin(wt) v_{0R} \sin(wt).$$

$$p(t) = i_0^2 R \sin^2(wt).$$

Average currents and voltages:

The average current $I_{\text{av}} = \lim\limits_{T\to\infty} \frac{1}{T}\int\limits_0^T i(t)dt$ is zero, since the current oscillates bewteen positive and negative values. The average voltage V_{av} across the resistor is also zero. Obviously, the average power dissipated in the resistor is not zero, since the average of $\sin^2(wt)$ is 0.5.

We define an **rms (root mean square) current** $I = \sqrt{\left(i^2\right)_{\text{av}}} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$ and an **rms voltage** $V = \sqrt{\left(v^2\right)_{\text{av}}} = \frac{v_0}{\sqrt{2}} = 0.707 v_0$. Then, Ohm's law takes the simple form $V_R = IR$. The **rms power** is $P = I^2 R = V^2/R$.

An inductor L in an AC Circuit:

From the loop rule, we conclude that $v - v_L = 0$. The voltage v_L is given by Faraday's law: $v_L = L\frac{di}{dt}$. By definition, $i(t) = i_0 \sin(wt)$, therefore $v_L(t) = i_0 Lw \cos(wt)$. The peak voltage is $v_{0L} = i_0 wL$ We see that the current and voltage and phase are 90° **out of phase.**

By analogy with Ohm's law, we define the **reactance** $X_L = wL$. Then $v_{0L} = i_0 X_L$ or $V_L = IX_L$. Note that the reactance increases linearly with increasing frequency.

The instantaneous power dissipated in the inductor is $p(t) = i(t)v_L(t)$. Thus $p(t) = i_0 \sin(wt) i_0 Lw \cos(wt)$. Therefore, the average power dissipated is zero, since $\sin(wt)\cos(wt)$ is zero on average.